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MODEL SCALING RULES FOR TURBOJET AND GAS TURBINE COMBUSTION CHAMBERS

by

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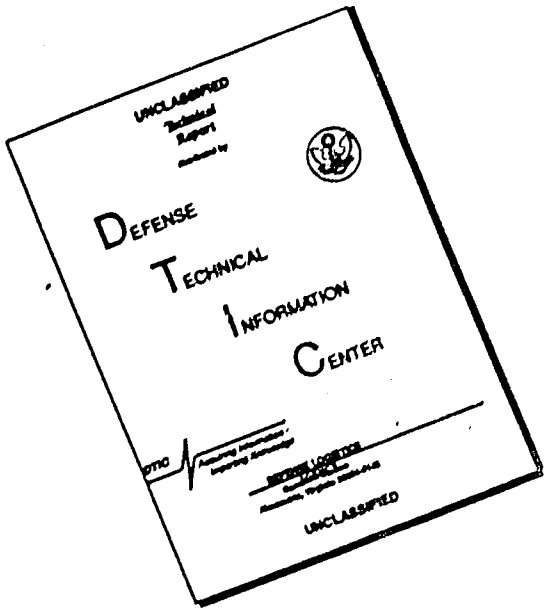

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MODEL SCALING RULES FOR TURBOJET AND GAS TURBINE COMBUSTION CHAMBERS

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ABSTRACT

→Many rules for scaling turbojet and gas turbine combustion chambers have been developed in the past ten years, but none has been accepted as a general rule. This paper describes various methods in order to find possible scaling rules for such a combustion system. A scaling theory with respect to nonhomogeneous combustion of droplets under jet flow is suggested primarily to check various existing model scaling methods. On the basis of the suggested theory, the author finally proposes some new scaling rules and also offers conclusions for reference use in future studies.( )

MODEL SCALING RULES FOR TURBOJET AND GAS TURBINE COMBUSTION CHAMBERS

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ABSTRACT

Many rules for scaling turbojet and gas turbine combustion chambers have been developed in the past ten years, but none has been accepted as a general rule. This paper describes various methods in order to find possible scaling rules for such a combustion system. A scaling theory with respect to nonhomogeneous combustion of droplets under jet flow is suggested primarily to check various existing model scaling methods. On the basis of the suggested theory, the author finally proposes some new scaling rules and also offers conclusions for reference use in future studies.

I. Introduction

In order to keep abreast of rapid developments in jet engines and gas turbines in civilian use, design and testing of combustion chambers have obviously become very important problems. Since combustion is a complex process, most combustion chambers designed in the past were based on experience alone. Construction of a high-performance prototype generally requires a series of experiments and adjustments and wastes large amounts of manpower and materials. This is considered uneconomical as far as savings in production and promotion of design technology are

concerned. Thus, the search for an applicable rule for combustion chamber scaling has become a practical problem.

Progress has been made in studies on combustion chamber scaling during the past ten years, with scaling theories and rules continually published[1]-[9]. The existing rules can be classified fundamentally into two categories. In the first category, actual combustion chambers are treated as a homogeneous gas reaction system, as for example the model scaling rules developed on the basis of turbulent combustion velocity theories[5][6] and those derived from G. Damkohler's five similarity criteria[3][4][8]. Combustion characteristics of liquid fuels are disregarded under this category, and only empirical correction indices such as experimental indices[5][6] related to the assumed order of reaction and mixing factors are introduced in the final analytical results. The second category is based principally on combustion of droplets in jet flow, with careful attention to combustion characteristics of liquid fuel and to scaling requirements for gas reaction, as for example the scaling rules described by Stewart[1] and by Herbert[7], those[2] falling between Stewart's and Herbert's, and those established purely by supposition through experimentation[9]. To sum up, there are numerous forms of existing scaling rules, which differ greatly from one another. It has not yet been possible to reach a unanimous viewpoint and understand their inherent connections. Thus, how to classify the existing rules, whether or not there are other possible schemes, and how to investigate these schemes are worth further investigation.

The key approach used in this paper to investigate model scaling laws controlled by droplet combustion under jet flow is through the similarity and combustion theories, because the processes in the combustion chambers of most jet engines and fuel-burning gas turbines, with the exception of those in a few vaporization-type and gas-burning combustion chambers, are controlled by thermal diffusion in droplet jet flow. Their working characteristics are closely related and very sensitive to atomization, mixing, vaporization, and combustion of liquid fuel. Much of the literature, including [7] [8], reports that various working conditions of an actual nozzle can be simulated by using a series of small nozzle models, and this is an obvious example.

II. Theoretical Data for Model Scaling

Combustion of liquid fuel is characterized by the following component processes: atomization, mixing, vaporization, and combustion. In order to make the actual combustion process and that in a model similar, in addition to the necessary geometric similarity in combustion chambers and their major components (including sprayers and flame stabilizers), similarities must be obtained to satisfy as far as possible these component processes and also the monodromy conditions of combustion systems. As for the internal heat-transfer and combustion gas cooling problems, generally speaking they are relatively easy to solve, and thus further discussion is unnecessary.

1. Atomization

To make the atomization processes in two similar combustion chambers identical, the same law governing the size and distribution of droplets in both combustion spaces must be applied. Fineness of atomization in jet flow can be designated by the mean droplet diameter d_s. According to dimensional analysis[10][11], the dimensionless mean droplet diameter can be expressed by the following dimensionless functional relation:

$$\frac{d_s}{d_j} = f(Re_j,\ We_j,\ \frac{\rho_l}{\rho_g},\ \frac{\mu_l}{\mu_g},\ \frac{v_l}{v_g},\quad ,\ \frac{l_1}{d_j},\ \frac{l_2}{d_j},\ \frac{l_3}{d_j}\ \ldots\ldots), \tag{1}$$

where d_j is the fuel nozzle orifice diameter; l_1, l_2, l_3, etc. are geometric nozzle dimensions; $Re_j = \frac{\rho_l v_l \cdot d_l}{\mu_l}$ and $We_j = \frac{\rho_l v_l^2 d_l}{\sigma}$ are Reynolds number and Weber number, respectively; ρ, μ, v, and σ represent density, viscosity, velocity, and surface tension coefficients, respectively, with subscripts l and g to denote liquid and gas; and θ is the injection angle of jet flow. The injection angle is related primarily to the structural parameters of the nozzle[10]. Although θ is influenced slightly by the combustion pressure and fuel injection pressure, these effects can be disregarded in approximation. Equation (1) for two geometrically similar nozzles can be written as

$$\frac{d_s}{d_j} = f(Re_j,\ We_j,\ \frac{\rho_l}{\rho_g},\ \frac{\mu_l}{\mu_g},\ \frac{v_l}{v_g}). \tag{2}$$

The applicable function form of the above equation has not yet been fully developed. If the same fuel and the same inlet temperature are taken, then the quantitative relationship comparatively suitable for a centrifugal

nozzle is

$$\frac{d_s}{d_j} = (a + bRe_j)We_j^{-\frac{1}{3}}\left(\frac{\rho_l}{\rho_g}\right)^{\frac{1}{5}}, \tag{3}$$

where a and b are coefficients. According to reference [12], a = 23.5 and b = 0.0004. This shows that the Reynolds number for jet flow has only a slight effect on $\frac{d_s}{d_j}$. Since the effect of velocity ratio $\frac{v_l}{v_g}$ is not involved in the equation, it is desirable to maintain the same ratio value in model scaling.

The penetration depth of droplets in jet flow is also an important index of atomization. Theoretical analysis shows that penetration depth S of droplets is directly proportional to the square of diameter d_s and injection velocity v_l. Thus the approximate quantitative equation[1][7] is

$$S \propto d_s^{1.8}\rho^{-0.14}v_l^{0.9}\mu^{-0.055}. \tag{4}$$

2. Mixing

Mixing of fuel and air in the combustion chamber of a gas turbine is through turbulent diffusion. Using a combustion system with the same kind of fuel and same temperature, similarity in mixing is obtained under the following necessary condition:

$$Re_g = \text{constant}, \tag{5}$$

where $Re_g = \frac{\rho_g v_g L}{\mu_g}$ is the Reynolds number of airflow. L is the scaling dimension of the combustion chamber.

The combustible mixture system formed by droplets existing in both liquid and gas phases satisfies only the equivalent Re_g condition, and

is inadequate to warrant mixing similarity, which includes similarity in final mixing results, i.e., similarity in the concentration fields. Thus, the supplementary condition is

$$f = \text{constant} \cdot \frac{Cv_j \cdot d_j^2 \rho_j}{v_g L^2 \rho_g} = \text{constant}, \tag{6}$$

and

$$f' = \text{constant}, \tag{6a}$$

where f is the overall fuel-to-air ratio and f' is the fuel-to-air ratio at an arbitrary point in the combustion space. Notation C in equation (6) denotes the flow coefficient of a fuel nozzle. The flow coefficient obtained by dimensional analysis is

$$C = f(Re_j). \tag{7}$$

The applicable quantitative relationship[15] may be taken as

$$C = \alpha Re_j^{-\frac{1}{9}}. \tag{7a}$$

To satisfy the conditions in equation (6a), in addition to similarity in configurations and arrangements, the ratio of the nozzle design dimension to the combustion chamber scaling length must be constant, or

$$\frac{dj}{L} = \text{constant}. \tag{8}$$

3. Vaporization

Vaporization of droplets in a combustion chamber is a very complex process, because droplets change speed and temperature constantly after injection from the nozzle. At the same time, external conditions and influencing factors also vary constantly with droplet movements. In

approximation treatment, however, droplet vaporization can be taken as a process consisting of a nonsteady stage and a steady stage.

Droplets move with constantly declining speeds and rising temperatures during the nonsteady period, as full entry into the main stream has not yet been attained and flame has not yet started nearby. Thus, droplets undergo pure vaporization during this period under a forced convective condition.

When droplets enter the main stream, their velocity relative to that of the main stream is taken as approximately zero. After passing through the nonsteady period, droplets have gained a surface temperature to reach thermal equilibrium for steady vaporization. Entry into the hot combustion region with flame nearby also enables the droplets to reach a state for steady vaporization as well as for combustion.

As for temperature field similarity, the principal similarity criterion governing nonsteady vaporization in combustion systems producing reactants of the same composition and characteristics should be the droplets' own Reynolds number Re_d. In order to make nonsteady processes similar, the same Reynolds number must be used for average droplets in two similar jet flows, or

$$Re_d = \frac{\rho_g v_g d_g}{\mu_g} = \text{constant.} \tag{9}$$

It is generally believed that the nonsteady vaporization period is much shorter than the steady combustion period, and this is especially true for heavy fuels. Nonsteady vaporization becomes relatively more important[16] in combustion of droplets under high pressure.

4. Steady Combustion

Vaporization and combustion occur simultaneously under a steady combustion condition. In order to make combustion processes similar, the ratio of combustion time t_b to the retardation time t_r of droplets in the combustion chambers must be kept constant, or

$$\frac{t_b}{t_r} = \text{constant.} \tag{10}$$

According to a series of single-drop combustion tests and theoretical analyses [17]—[20], the law governing combustion of droplets is

$$d^2 = d_3^2 - Kt, \tag{11}$$

where d is the droplet diameter and K is the vaporization constant. Recent literature [21] [22] reports that the above equation is also applicable to various droplet sizes and comparatively homogeneous distribution of droplets in jet flow and to combustion with multiple dripping. Under the last condition, d should be the mean droplet diameter and K the mean vaporization constant. According to combustion theories [17] [18], vaporization constant

$$K \propto \frac{\lambda_g}{C_g \rho_l} \ln(1+B), \tag{12}$$

where λ_g and C_g denote the coefficient of thermal conductivity and specific heat of gas, respectively, and B is a dimensionless transfer parameter. K approaches the same value under identical fuel and temperature conditions. The effect of working pressure p_g in a combustion chamber on vaporization constant K is still indeterminate. According to the only experimental report [20],

$$K \propto p_g^{\frac{1}{4}}. \tag{13}$$

According to the nonsteady combustion theory [16] for combustion of droplets in a high-pressure combustion chamber,

$$K \propto p^{-\frac{1}{2}} . \tag{14}$$

Since the pressure effect on combustion velocity or combustion time is still unclear, it is desirable to disregard this factor in model scaling.

If equation (11) and $t_r = \frac{L}{v_g}$ are substituted in equation (10), then

$$\frac{t_b}{t_r} = \frac{v_g d_s^2}{KL} = \text{constant.} \tag{15}$$

5. Stability of Combustion

Flame blowoff characteristics of a combustion chamber often serve as an important index of simulation, and are considered in scaling scheme selection. A flame stabilizer is generally installed in the combustion chamber of a gas turbine. Since the temperature in the combustion gas circulation region in the rear section of a flame stabilizer is rather high, it is believed that complete atomization of droplets occurs in this region and the following results can be obtained from a homogeneous gas mixture system [18]:

$$V_{bo} \propto p^{n-1} L, \tag{16}$$

where V_{bo} is the critical flame blowoff velocity and n is the assumed order of reaction. Equation (16) shows that the blowoff velocity can be lowered in a reduced scale model with the working pressure remaining unchanged. Under such a condition, it is difficult to obtain an airflow velocity in a model higher than that in a real object without the aid of

a flame stabilizer. In addition, the model has a very narrow working simulation range. To make the blowoff characteristics similar, it is necessary to satisfy

$$\frac{V_{bo}}{V_g} = \text{constant.} \tag{17}$$

Since combustion is a very complex process, it is still impossible at present to derive an appropriate similarity criterion strictly by resorting to a system of equations describing a process. Similarity in the actual performance of important component processes can be obtained only on the basis of scaling requirements.

III. Analysis of Existing Model Scaling Schemes

The writer analyzed and calculated the existing scaling schemes on the basis of the theory suggested above. Analysis was carried out in two major steps. Step I was checking of the necessary conditions of dimensionless similarity parameters in the processes, and Step II was comparison of actual principal working parameters or important technical indices with those of models. Calculation results are tabulated in Table 1.

Scheme I in Table 1 is actually the so-called Stewart scaling rule. Calculation results in the table show that various individual dimensionless similarity parameters of that scheme in atomization are rather unsatisfactory, as We_j and $\frac{\rho_l}{\rho_g}$ are not identical, particularly the severe effect of Weber number We_j for jet flow on the breakdown of liquid in the

later period. The mean droplet diameter increases with the declining We_j value as the actual dimensions are reduced in scale. These two opposing factors, however, nearly cancel each other as combustion pressure increases. Thus, the final results in atomization, i.e., $\frac{d_{sM}}{d_{jM}}\Big/\frac{d_{sH}}{d_{jH}} = m_l^{0.02}$ and $\frac{S_M}{L_M}\Big/\frac{L_H}{L_H} = m_l^{0.13}$ are still able to attain approximation. Mixing in Scheme I is comparatively satisfactory. First, the overall fuel-to-air ratios are identical and the Reynolds numbers for air flow are also identical. Secondly, although the concentration fields cannot be kept strictly similar due to nonconformity in geometric scaling of the nozzle and the combustion chamber, the condition is not considered serious. Moreover, the nozzle orifice diameter has not been reduced to scale, and this practice makes construction even easier. The most serious drawback of Scheme I is that dimensionless combustion time $\frac{t_b}{t_r}$ in the model and in a real object cannot be identical, as the model has a much longer combustion time. As a result, similarity in combustion of droplets cannot be obtained, for combustion efficiency in the model may be higher. Another drawback of Scheme I is that the blowoff velocity in the model is lower than that in the real object, because the effect induced by a reduction in flame stabilizer geometric dimensions is greater than the effect induced by an increase in pressure. Thus, the blowoff characteristics are also dissimilar. This scheme, however, exhibits superior simulation quality as compared with the other schemes.

Scheme I has been supplemented by Herbert and Bamford[7], who believe that similarities in atomization and in combustion can be obtained

Table 1. Analysis of existing scaling schemes for gas turbine combustion chamber

Scheme		Principal scaling rule	Necessary conditions for dimensionless parameters in the process		
			Atomization		
			1	2	3
			Relative Reynolds number for jet flow	Relative Weber number for jet flow	Relative velocity ratio
I		1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; 5. pL=constant; 6. $v_l \propto v_g$=constant.			
II		1. T_o=constant; 2. identical fuel characteristics; 3. pL=constant; 4. v_g=constant; 5. v_l/v_g=constant; 6. $d_s \propto \sqrt{L}$; 7. geometric similarity in combustion chambers and nozzles; $d_j/l \neq$constant.			
III	a	1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; $d_j/L \neq$constant; 5. $p^{n/2}L$=constant, n=1.7; 6. $v_j \propto \sqrt{l}$; 7. $d_j \propto L^{0.29}$.			
	b	1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; $d_j/l \neq$constant; 5. $p^{n/2}l$=constant, n=1.7; 6. $v_l \propto \sqrt{L}$.			

IV		1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; $d_j L \neq$constant; 5. p_g=constant; 6. $v_g \propto L$; 7. v_l=constant (fuel supply pressure unchanged).			
V	a	1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; 5. $\rho_g v_g$=constant; 6. $d_{gM}/d_{gH}=\sqrt{\frac{\rho_M}{\rho_H}\frac{L_M}{L_H}}$			
	b	1. T_o=constant; 2. identical fuel characteristics; 3. f=constant; 4. geometric similarities in combustion chambers and nozzles; d_j/L=constant; 5. pv_g=constant; 6. $v_l \propto L^{1/4}$.			
Remarks		Note 1: Blowoff velocity V_{bo} is calculated based on $V_{bo} \propto p^{n-1}L$, where order of reaction n = 1.7. Note 2: Effect of Re number on vaporization constant K is disregarded when relative dimensionless combustion times for Schemes IV and V are calculated. If this effect is considered, then discrepancies in calculation will be much greater. Note 3: In calculating power loss from air supply, inlet temperatures and compressor efficiencies in both the model and the real object are assumed to be identical. Note 4: Notation $m_l = L_M/L_H$ in the table indicates geometric scaling, where subscripts M and H denote the model and real object, respectively.			

Necessary conditions for dimensionless parameters in the process

Atomization				Mixing			Vaporization and combustion	
4	5	6	7	8	9	10	11	12
Relative density ratio	Relative viscosity ratio	Relative dimensionless penetration depth	Relative dimensionless droplet ratio	Relative fuel-to-air ratio	Relative Reynolds number for air flow	Relative geometry ratio	Relative droplet Reynolds number	Relative dimensionless combustion time

Flame blowoff	Principal parameter ratios in the system					
13	14	15	16	17	18	19
Relative dimensionless blowoff velocity	Jet flow velocity	Nozzle orifice diameter	Combustion pressure	Airflow velocity	Droplet diameter	Blowoff velocity

20 Airflow	21 Power loss from air supply	Source of original data
		[1]
		[7]
		[2]
		[8]
		[9]

by using a centrifugal nozzle in the combustion chamber, as the mean diameter of droplets injected by the nozzle in a model or in a real object must be directly proportional to the square root of the combustion chamber design dimension. Data for Scheme II are obtained from Scheme I revised by supplementation. Because of the effect induced by the coefficient of nozzle flow on droplet size, which is taken into account by the theory suggested in this paper, and of the variation in relationship between pressure and combustion velocity at an index of $\frac{1}{4}$, the fuel-to-air ratios f thus obtained are not identical, and the dimensionless combustion times are also nonidentical. To sum up, the results obtained do not differ greatly from those of Scheme I; but if the pressure has no effects on combustion velocity as assumed in reference [7], then similarity in combustion can be closer than that shown in Scheme I.

Scheme III is equivalent to the scheme described by Way[2], who assumes reaction in a combustion chamber on the basis of homogeneous gas reaction. According to this assumption, the relationships of the working pressure in a combustion chamber with airflow velocity and design dimensions for scaling can be obtained by Damkohler's five similarity criteria. Scheme IIIb in the table is obtained on the basis of Way's assumption by taking $n = 1.7$ to calculate various relative data. Scheme IIIa is obtained on the basis of the nonhomogeneous drop control theory, using Way's original data in calculation. When the theory suggested in this article is applied, the relative dimensionless combustion time will not be equal to 1. In other words, the same combustion

time cannot be maintained when the process is controlled by a non-homogeneous reaction, according to the Way scaling rule. It must be pointed out that the Way scaling method uses the injection velocity as the design velocity for calculating retardation time, and the writer believes that the airflow velocity should be used as the design velocity. Calculation results in Scheme III show that similarity in atomization varies greatly from those in Schemes I and II; not only is it impossible to keep dimensionless parameters for the process identical, especially velocity ratio $\frac{v_l}{v_g}$, but final results for atomization also differ greatly. The model has a lower combustion efficiency, as the dimensionless droplet diameter is much larger and the dimensionless combustion time shorter than the values in a real object. Thus, according to theoretical analysis, Scheme III is inferior to Schemes I and II.

The principal scaling rule for Scheme IV is suggested by Lebedev[8]. In fact, this rule is merely Damkohler's first similarity criterion $D = \frac{t_r}{t_j}$ (where t_j is the reaction time) and the Re number is assumed for a "self-scaling" [Translator's note: May mean dynamically similar or self-similar] condition (it is doubtful whether or not the experimental condition reported by that paper actually attained a self-scaling condition). This scheme is thus based entirely on homogeneous gas reaction without taking mixing factors into sufficient account. Reference[8] does not describe the experimental scaling conditions for a nozzle fully, but only briefly mentions invariable supplied oil pressure and reduction in dimensionless nozzle orifice diameter with the geometric characteristics

of a nozzle in the model remaining unchanged. The writer made calculations for such a condition on the basis of his own theory. From the calculations it is revealed that if initial conditions are maintained, the nozzle orifice diameter in the model is reduced $(L_M/L_H)^{1.59}$ times as compared to that in a real object. Although the mean droplet diameter is reduced L_M/L_H times from its actual value through geometric scaling, the dimensionless droplet diameter still varies greatly with the reduced nozzle orifice diameter; thus making it impossible for atomization processes to reach similarity. As the retardation time of droplets in the real object and that of droplets with reduced diameter in the model are the same, the relative dimensionless combustion time, not to mention similarity in various mixing parameters, also differs greatly. The writer believes that the Lebedev gas scaling rule at p_g = constant and $\bullet \propto L$ is groundless, because fineness of atomization was not checked in the original article for satisfactory explanation of rapid droplet varporization into gas and rapid mixing of the gas with air in a model or under actual conditions; thus the role of the similarity criterion controlling atomization is rendered unimportant.

Briskin[9] applied a scaling rule in tests conducted under the following requirements: $p_g v_g$ = constant, $\frac{d_{sM}}{d_{sH}} = \sqrt{\frac{p_M}{p_H} \cdot \frac{L_M}{L_H}}$, and assumption of Re_g in the self-scaling region of a combustion chamber. He failed, however, to check the droplet diameter before combustion tests were conducted. Nozzle scaling conditions were not clearly described and the supplied fuel pressure at 40 kg/cm^2 was only vaguely mentioned. Scaling conditions were suggested, but not theoretically explained in detail.

Finally, he considered that working similarity in combustion chambers can be obtained with conditions verified by tests. Another noteworthy phenomenon was the comparatively narrow working range obtained in the tests. The coefficient of excess air α_1 varied only between 1.3 and 1.7. The reason for such a narrow working simulation range is not explained. According to the Briskin scaling conditions (assuming v_l = constant and $d_{jM}/d_{jH} = L_M/L_H$), the calculation results shown in Scheme Va of Table 1 are obtained. The scaling method is characterized by having nearly identical dimensionless combustion times. Atomization and mixing results as well as various similarity parameters are unsatisfactory. The blowoff characteristic in the model is far inferior to the actual condition. This may be the main reason for the narrow test range; inability to keep the processes in both systems similar during changes in operation may also limit the working range in tests. Scheme Vb is Va revised by assuming that the injection velocity is $v_l \propto L^{1.8}$. The pressure ratios governed by the law of change thus obtained may be closer to Briskin's test data.

IV. Search for Possible Scaling Schemes for Combustion Chambers on the Basis of Theories Governing Nonhomogeneous Droplet Combustion

It can be seen from the theoretical analysis in the preceding sections that the optimum scheme among the existing scaling schemes is the one using pL = constant as a scaling rule. For further discussion of scaling schemes and dependence relationships between parameters under a given necessary scaling condition, investigation of the following

aspects is made on the basis of the theory suggested in Section II.

According to the theoretical data for controlling droplet combustion in jet flow, the writer takes 1) fuel inlet temperature T_o = constant; 2) identical fuel characteristics; 3) fuel-to-air ratio f = constant; 4) geometric similarities in combustion chambers and nozzles, but d_j/L not necessarily identical; 5) Re_g = constant; and 6) dimensionless combustion time t_b/t_r = constant and other conditions to be considered as necessary scaling conditions for the problems under discussion. The first three conditions simplify theoretical analysis greatly, because many effects induced by physical parameters can be largely eliminated when the same fuel (including f = constant) and identical reaction temperature conditions are used in a model and a real object. Geometric similarity of nozzles is especially necessary for scaling under condition 4; otherwise difficulties are involved in obtaining identical atomization characteristic parameters such as injection angle θ and distribution of various droplet sizes. To make requirements less rigid in construction of nozzles for a model, d_j/L need not necessarily be a constant. In scheme selection, suppose the injection angles are identical, the Re_g numbers of the principal similarity criterion controlling mixing are equivalent, and the mean dimensionless droplet diameters and penetration depths in both systems can be made as nearly identical as possible. Then similarity in concentration distribution will not be seriously affected, even if the geometric dimensions of the nozzle and combustion do not strictly concur. The most important similarity criterion is t_b/t_r = constant, because it determines similarity in steady vaporization and combustion of droplets in jet flow.

In searching for possible schemes in addition to these five necessary scaling conditions, the atomization effects must be kept as closely identical as possible, first by making d_s/d_j = constant and then by giving more attention to the velocity ratio $\frac{v_l}{v_g}$, because the latter plays an important role in atomization concerning the mean droplet diameter. Nonsteady vaporization is not a deciding factor, as its duration is very short as compared with the steady state. The blowoff characteristic is a comparatively important simulation index under certain conditions, but can be disregarded when certain equipment is used. Nevertheless, the blowoff limit may often become a parameter determining the selection of a scheme, because many schemes have various equally satisfactory indices, but a low dimensionless blowoff velocity. These schemes cannot be applied unless a special flame stabilizer is installed.

A system of fundamental equations for solving various problems can be developed on the basis of the above conditions and the theory in Section II.

1) Overall fuel-to-air equation [from equation (6)]:

$$\left(\frac{d_{jM}}{d_{jH}}\right)^{9/7} = \frac{p_M}{p_H}\left(\frac{v_{lM}}{v_{lH}}\right)^{-8/9}\left(\frac{V_{gM}}{V_{gH}}\right)\left(\frac{L_M}{L_H}\right)^2. \tag{20}$$

2) Atomization equation [from equation (3)]:

$$\frac{d_{sM}}{d_{sH}} = \left(\frac{d_{jM}}{d_{jH}}\right)^{2/3}\left(\frac{v_{lM}}{v_{lH}}\right)^{-2/3}\left(\frac{p_M}{p_H}\right)^{1/3}. \tag{21}$$

3) Equation for combustion of droplets in jet flow [obtained from equations (10) and (11)]:

$$\frac{d_{sM}}{d_{sH}} = \left(\frac{L_M}{L_H}\right)^{1/2}\left(\frac{v_{gH}}{v_{gM}}\right)^{1/2}\left(\frac{P_M}{P_H}\right)^{x}, \tag{22}$$

where x is an index determining the effect of combustion pressure on combustion velocity or vaporization rate. If we let $K \propto p^{1/4}$ from expression (13), then x = 1/8 in the above equation. If pressure has no effects on vaporization rate K, then x = 0. If we let $K \propto p^{-1/3}$ as in expression (14) for droplet combustion under high pressure, then x = -1/6. In the writer's opinion, pressure must have effects on vaporization velocity, particularly when the working pressure in a real object differs greatly from that in a model. When the working pressure exceeds the critical pressure during droplet vaporization, the vaporization mechanism may undergo a pronounced change. Two separate results are shown in the following calculations by taking x as 1/8 and as 0.

4) Mixing equation (Re_g = constant):

$$\frac{v_{gM}}{v_{gH}} = \frac{P_H}{P_M} \cdot \frac{L_H}{L_M}. \tag{23}$$

Six parameters, i.e., $\frac{d_{jM}}{d_{jH}}$, $\frac{v_{jM}}{v_{jH}}$, $\frac{v_{gM}}{v_{gH}}$, $\frac{P_M}{P_H}$, $\frac{d_{sM}}{d_{sH}}$, and $\frac{L_M}{L_H}$, are included in the four fundamental equations. If (L_M/L_H) is taken as a known parameter, one of the five unknowns must be taken as an independent variable, leaving the variation relationships of the other four dependent variables to be determined.

Thus, a set of function relations can be derived from equations (20) and (23) at x = 1/8 as follows:

$$\frac{P_M}{P_H} = \left(\frac{L_M}{L_H}\right)^{-0.72}\left(\frac{v_{jM}}{v_{jH}}\right)^{-1.19}, \tag{24}$$

$$\frac{v_{pM}}{v_{jH}} = \left(\frac{L_M}{L_H}\right)^{-0.21}\left(\frac{v_{lM}}{v_{lH}}\right)^{1.19}, \tag{25}$$

and

$$\frac{d_{sM}}{d_{sH}} = \left(\frac{L_M}{L_H}\right)^{0.5}\left(\frac{v_{lM}}{v_{lH}}\right)^{-0.75}, \tag{26}$$

$$\frac{d_{jM}}{d_{jH}} = \left(\frac{L_M}{L_H}\right)^{9/17}\left(\frac{v_{lM}}{v_{lH}}\right)^{-8/17}, \tag{27}$$

.....

....., etc.

Similarly, at $x = 0$ when the combustion time is not affected by pressure, another set of function relations is obtained as follows:

$$\frac{P_M}{P_H} = \left(\frac{L_M}{L_H}\right)^{-0.925}\left(\frac{v_{lM}}{v_{lH}}\right)^{-1.4}, \tag{24a}$$

$$\frac{v_{gM}}{v_{gH}} = \left(\frac{L_M}{L_H}\right)^{-0.08}\left(\frac{v_{lM}}{v_{lH}}\right)^{1.4}, \tag{25a}$$

$$\frac{d_{sM}}{d_{sH}} = \left(\frac{L_M}{L_H}\right)^{0.52}\left(\frac{v_{lM}}{v_{lH}}\right)^{-0.7}, \tag{26a}$$

and

$$\frac{d_{jM}}{d_{jH}} = \left(\frac{L_M}{L_H}\right)^{9/17}\left(\frac{v_{jM}}{v_{lM\,[sic]}}\right)^{-8/17}, \tag{27a}$$

.....

....., etc.

Injection velocity is used as a variable in calculation by this set of equations, and the complete results are shown in Tables 2A and 2B, where the injection velocity varies from $v_j \propto L$ to $v_j \propto L^{1/2}$. Table 2A shows that dimensionless droplet diameter and penetration depth ratios are close to 1

Table 2A Corresponding change conditions of scaling parameters at various fuel injection velocities

(Principal scaling conditions are: inlet temperatures identical; fuel characteristics identical; f = constant; combustion chambers and fuel injection nozzles geometrically similar, respectively; Re_g = constant; t_b/t_r = constant, and taking $K \propto p^{-1/4}$.)

Scheme	Fuel injection velocity	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	19	20
1 2 3 4 5 6				
7	v_l =constant			
8	p=constant $v_l \propto v_g$=constant		See Table 1	
9 10 11 12 13 14				

Remarks

1. In calculation for this table, the effect of working pressure in the combustion chamber on droplet combustion time is treated by assuming index $(-\frac{1}{4})$ for variation relationship.

2. The effect of velocity ratio v_l/v_g on dimensionless droplet diameter is disregarded in calculation, but the latter is taken into account in comparison of schemes.

3. For sake of comparison, the Stewart scheme, or Scheme 8, is introduced in the table, but it does not concur with the t_b/t_r = constant condition.

Table 2B Corresponding change conditions of scaling parameters at various fuel injection velocities

(Principal scaling conditions: inlet temperatures identical; fuel characteristics identical; f = constant; combustion chambers and fuel injection nozzles geometrically similar, respectively; Re_g = constant; t_b/t_r = constant; and taking K independent of pressure.)

Scheme	Fuel injection velocity	1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18	19	20
1 2 3 4 5 6				
7	v = constant			
8	p_l=constant $v_l \propto v_g$=constant, $d \propto \sqrt{L}$		See Table 1	
9 10 11 12 13 14				

Remarks

1. In calculation for this table, working pressure in the combustion chamber is assumed to have no effects on droplet combustion time; that is, vaporization constant K is independent of pressure.

2. The effect of velocity ratio v_l/v_g on dimensionless mean droplet diameter is disregarded in calculation, but the latter is discussed in comparison of schemes.

3. For sake of comparison, the Herbert and Bamford scheme, or Scheme 8, is introduced in the table. Various data of that scheme are obtained based on theoretical analysis in this paper.

when v_l varies from $L^{1/8}$ to $L^{-1/8}$. The velocity ratios of the model and the real object do not exceed 0.85 at $L_M/L_d = \frac{1}{2}$. The nozzle orifice diameter relative to the injection velocity changes between $(L_M/L_H)^{0.47}$ and $(L_M/L_H)^{0.59}$. Therefore, it is possible to obtain similarity in combustion conditions, if this region is used for scaling. Scheme 7, in which v_l = constant, happens to fall in this region. Scheme 8 (Stewart scheme) is also inserted in Table 2A especially for comparison. From a theoretical viewpoint, the Stewart scheme is not a unique possible scaling scheme. The next scheme, however, uses the same fuel injection and air inlet velocities to eliminate the effect of velocity ratio on atomization results, and its blowoff characteristic is superior to that of Schemes 5—10 in Table 2A. As noted from the blowoff characteristic, it is a disadvantage to use an injection velocity in the model higher than that in the real object; otherwise, the applicability of all the above scaling schemes cannot be ensured.

Curves showing the variation of relative dimensionless similarity parameters and of principal scaling indices with injection velocity at a geometric scale of $m_l = \frac{L_M}{L_H} = \frac{1}{2}$ are plotted in Fig. 1 and 2.

Fig. 1 shows that all dimensionless similarity parameters obtained at the $\frac{v_{lM}}{v_{lH}} = 0.34—1.09$ variation range (equivalent to the $v_l \propto L^{-1/8}$ region) are rather satisfactory. The most seriously impaired is the blowoff characteristic. Blowoff velocity in the model under the above scaling conditions is always lower than that in a real object; thus it is difficult to make (V_{bo}/V_g) a constant unless the air inlet velocity is greatly reduced. It is possible only by making $\frac{v_{lM}}{v_{lH}}$ equal to 0.8, or

equivalent to about $v_l \propto L^{\frac{1}{4}}$. As noted, the optimum scaling region should be in the $v_l \propto L^{\frac{1}{4}} - v_{l\,H} = v_{l\,M}$ range. Various important scaling indices relative to this variation region may be obtained from Fig. 2 or Table 2A. The dotted lines in Fig. 1 and 2 indicate v_l in the variation region from $v_l \propto L^{\frac{1}{4}}$ to $v_l \propto L^{-\frac{1}{4}}$. Parameters deviate greatly outside this region, with similarity in processes seriously impaired and, especially, reduction of blowoff velocity in the model. As a result, similarity in combustion cannot be realized unless a flame stabilizer is installed in the model. Since it is still difficult to evaluate blowoff characteristics in a combustion chamber quantitatively, the above analysis can be used only as a qualitative reference.

.Suppose pressure has no effects on vaporization rate (see Table 2B); conditions are then slightly different. First, with other important scaling conditions identical, a higher working pressure is to be applied to the model. Secondly, the optimum scaling scheme deviates slightly, but the region in which similarity of processes may be maintained remains at $v_l \propto L^{1/8} - v_l \propto L^{-1/8}$, and is optimum at variations between $v_l \propto L^{1/16}$ and $v_l \propto L^{-1/16}$.

Therefore, regardless of whether or not pressure has effects (not counting negative index effects) on combustion velocity, the optimum scaling scheme occurs in a region where injection velocity is a constant or is slightly lower than that of a real object. If scaling is carried out in or near this range, there is a possibility of obtaining satisfactory results. If the flame stabilization capability and simulation performance of a model with variable working conditions are to be improved, it is

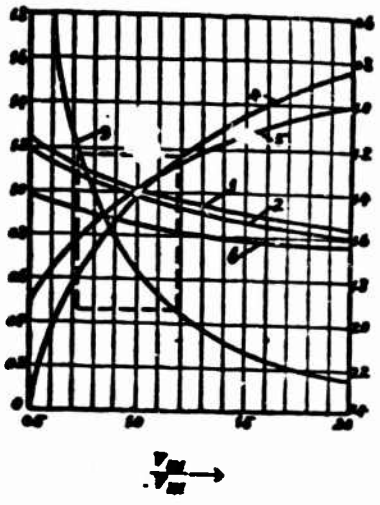

Fig. 1. Variation relationship between relative dimensionless parameters and fuel injection velocities at a geometric scale of 1 : 2

1 - Dimensionless droplet diameter ratio ($\bar{d}_s/d_j$); 2 - dimension penetration depth ratio ($\bar{S}/L$); 3 - dimensionless blowoff velocity ratio ($\bar{v}_{bo}/v_g$); 4 - dimensionless droplet Reynolds ratio ($\bar{R}e_d/d_j$); 5 - dimensionless nozzle geometry ratio ($\bar{d}_j/L$); and 6 - dimensionless velocity ratio ($\bar{v}_l/v_g$).

(Ordinates on the left are used for curves 1, 2, 3, and 6; ordinates on the right are used for curves 4 and 5)

Dotted lines indicate the variation region at injection velocities from $v_l \propto L^2$ to $v_l \propto L^{-4}$.

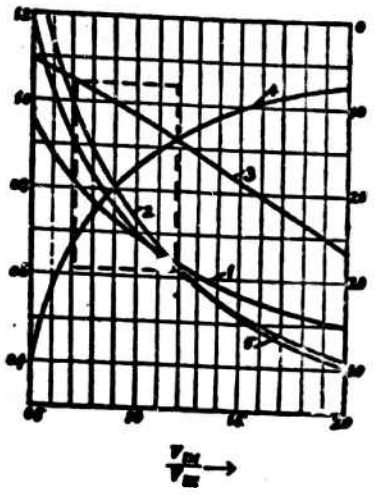

Fig. 2. Curves of principal scaling parameters in a model and a real object versus injection velocity ratios at a geometric scale of 1 : 2

1 - Nozzle orifice diameter (d_{jM}/d_{jH}); 2 - blowoff velocity (v_{boM}/v_{boH}); 3 - airflow velocity (v_{gM}/v_{gH}); 4 - combustion pressure (p_M/p_H); 5 - droplet diameter (d_{sM}/d_{sH}).

(Ordinates on the left are used for curves 1, 2, and 5; ordinates on the right are used for curves 3 and 4)

Dotted lines indicate the variation region at injection velocities from $v_l \propto L^{\frac{1}{2}}$ to $v_l \propto L^{-\frac{1}{4}}$.

necessary to select a lower injection velocity, a correspondingly reduced airflow velocity to match, and an appropriately higher working pressure. The latter, however, is disadvantageous to power loss from air supply. Under the above scaling conditions, the blowoff ratio does not vary with the changing injection velocity, but power loss increases with increasing combustion pressure. Therefore, regardless of whether scaling is carried out according to the rule with pL = constant or to the rule suggested in this section, the pressure in a model is to be higher than that in a real object. This is applicable only to models having a low initial working pressure, as in the case of scaling an aircraft combustion chambers; otherwise, elaborate air supply equipment is needed.

V. Investigation of Approximate Scaling Schemes under Isobaric Condition

The application of a high-pressure scheme for scaling a large complicated gas turbine engine with high-compression ratio has the following two disadvantages:

1) The combustion pressure in the original engine is already high. If the working pressure in the model is further increased, then it is necessary to use a compressor having a higher compression ratio than the actual equipment, and this is disadvantageous to test conditions.

2) The effects of combustion pressure on vaporization and combustion are as yet unclarified at present, and test data on combustion under high pressure are still scarce. The vaporization and combustion

mechanisms may undergo a pronounced change when the working pressure in a model exceeds that in a real object.

On the basis of the above principles, it is best to use an isobaric scaling scheme (the same combustion pressure for model and real object). It is difficult, however, to obtain the same Reynolds number for airflow by applying the isobaric scaling method when geometric dimensions of the model are reduced. As analyzed above, the airflow velocity in the model may exceed the blowoff velocity permitted by the flame stabilizer. When scaling under the isobaric condition is carried out, leniency in mixing requirements is inevitable. In other words, Re_g cannot be maintained strictly identical, and the atomization criterion also loses its effectiveness.

Comparison of calculations shows that a rather satisfactory approximate scaling scheme under the isobaric condition is: 1) T_o = constant; 2) fuel properties identical, f = constant; 3) v_g = constant; and 4) t_b/t_r = constant. The corresponding important scaling indices are:

$$\frac{v_{jM}}{v_{jH}} = \left(\frac{L_M}{L_H}\right)^{0.21};\ \frac{d_{jM}}{d_{jH}} = \left(\frac{L_M}{L_H}\right)^{0.96} \approx \left(\frac{L_M}{L_H}\right);\ \frac{d_{sM}}{d_{sH}} = \left(\frac{L_M}{L_H}\right)^{0.5};\ \left(\frac{G_M}{G_H}\right) = \left(\frac{L_M}{L_H}\right)^2$$

$$\left(\frac{\bar{v}_j}{v_g}\right) = \left(\frac{L_M}{L_H}\right)^{0.21};\ \left(\frac{\bar{d}_s}{d_j}\right) = \left(\frac{L_M}{L_H}\right)^{-0.46};\ \left(\frac{\bar{S}}{L}\right) = \left(\frac{L_M}{L_H}\right)^{0.09};$$

$$\left(\frac{\bar{d}_j}{L}\right) = \left(\frac{L_M}{L_H}\right)^{-0.04};\ (\bar{Re}_g) = \left(\frac{L_M}{L_H}\right),\ \frac{v_{boM}}{v_{boH}} = \frac{L_M}{L_H};\ \ldots\ldots.$$

Suppose airflow velocity in a model is lower than that in a real object. Then the atomization, vaporization, and mixing conditions will

be less favorable. Further increase of the airflow velocity in a model will affect the blowoff condition even more seriously.

As noted, an isobaric scaling scheme is applicable only to high-flow engines having a high compression ratio. Re_g, even reduced in the model, can still remain in the self-scaling region, because the Reynolds number of the airflow in the real object is very high.

VI. Conclusions

1. The approximate scaling rule for a combustion chamber is not unique. Under conditions analyzed in Section IV, satisfactory approximation may be obtained with fuel injection velocities in the region from $v_j \propto L^{\frac{1}{4}}$ to $v_j \propto L^{-1/16}$.

2. Variation of fuel injection pressure and nozzle outlet diameter has a great effect on approximation in scaling. Thus, a scaling rule obtained by coincidence in a single test without clearly designated working conditions for the nozzle is undependable. Even the accuracy of an established scaling rule can be affected by a slight error in nozzle construction.

3. Among the existing scaling rules, a rule with pL = constant is the most suitable, particularly for scaling of jet engine combustion chambers.

4. Nothing prevents testing high-flow gas turbine engines having a high compression ratio with the approximate scaling scheme. Similarity in combustion conditions may be obtained by the scaling rule using p = constant, v_g = constant, $v_j \propto L^{1/5}$, and $d_j \propto L$ when the Reynolds number

for airflow in a real object exceeds 4×10^5. Low-pressure simulation tests conducted under low Reynolds numbers can only be used as a rough check on performance sensitivity.

5. The theoretical analytical method in this paper may be used to evaluate scaling materials qualitatively for reference in future studies.

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